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IS 12347 (1988): Analysis of Means - A Graphical Procedure
[MSD 3: Statistical Methods for Quality and Reliability]



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IS : 12347 - 1988

Indian Standard
ANALYSIS OF MEANS —
A GRAPHICAL PROCEDURE

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Indian Standard

ANALYSIS OF MEANS — A GRAPHICAL PROCEDURE

0. FOREWORD

0.1 This Indian Standard was adopted by the Bureau of Indian Standards on 3 March 1988, after the draft finalized by the Quality Control and Industrial Statistics Sectional Committee had been approved by the Executive Committee.

0.2 The control charts are useful in indicating the presence of an assignable cause (of non-random variability). The technique of control charts, however, does not provide an automatic corrective action in the way mechanical or electrical systems do. Instead, it gives a warning signal to the operator that he must take corrective on his machine or process to ensure maintenance of quality in further production. When the nature of the assignable cause and the appropriate adjustment of the process are not known, there is a need to design production studies (or experiments). However, the majority of experiments need professional guidance when they begin to analyze and interpret data arising from designed experiments using analysis of variance procedure. Consequently, there may be reluctance to plan

and carry out a more sophisticated experimental design.

0.3 The method of analysis described in this standard provides a simple analysis of design of experiments and effective method of presenting the findings through an extension of control chart techniques. This method deals directly with means (in contrast to the analysis of variance technique in which means are compared by taking ratios of mean squares). The underlying concepts of the two techniques, namely, analysis of means and analysis of variance are basically the same and the conclusions drawn are usually the same. However, some differences between two methods are indicated in the standard. Additional benefits of analysis of means are also outlined in the standard.

0.4 In reporting the result of a test or analysis, if the final value, observed or calculated, is to be rounded off, it shall be done in accordance with IS : 2-1960*.

*Rules for rounding off numerical values (*revised*).

1. SCOPE

1.1 This standard provides a study of the possible effects of the factor by using analysis of means techniques. The method involves carrying out graphical analysis of the data obtained from different sources/groups and thereby comparing the relative importance as well as the statistical significance of various factors. The technique of analysis of means is illustrated with the help of an example.

2. TERMINOLOGY

2.1 Assignable Cause — A cause of systematic variation.

2.2 Control Chart — A chart on which limits are drawn and on which are plotted values of any statistics computed from successive samples of a production. The control chart is used to investigate if a process may be considered to be under control.

2.3 Decision Lines — In a control chart, the limits below which (upper decision line) or above which (lower decision line), or the lines between which the statistic under consideration lies with an

assigned high probability when the process is under control.

2.4 Mean — The sum of values divided by their number.

2.5 Parameter, Population Parameter — A quantity used to describe distribution of a characteristic in the population (*see also* 2.6).

2.6 Population — The totality of items under consideration.

2.7 Range — The difference between the largest and the smallest observations.

2.8 Sample — One or more items taken from a lot (population) intended to provide information on the lot (population) or on the process which had produced it.

2.9 Sample Size — Number of items in a sample.

2.10 Standard Deviation — The positive square root of arithmetic mean of the square of deviations taken from their arithmetic mean.

3. BASIC CONCEPTS

3.1 One method of studying a scientific process is to hold constant all variables which are suspected

of contributing to the variability of the process, and then decide whether the resulting pattern of data represents a stable source, or whether the data gives evidence of 'unstability' (non-randomness). Frequently, an experimenter will expect his own scientific process to indicate stability when his known independent variables are held constant; he expects, also, that the data obtained under these constant conditions will support his expectation and indicate only the presence of random variability. However, it is a common experience to find that the data, obtained under these process conditions considered to be stable (in control), indicate evidence of important non-randomness. Such evidence, if recognized, can be the basis of scientific discovery; if not recognized, it can lead to incorrect conclusions.

3.2 A second approach to experimentation is to vary deliberately and simultaneously, in a pre-planned experimental pattern or design, different factors suspected of contributing to variation of the process. A standard method of comparing effects observed from such experiments is 'analysis of variance'.

3.3 The method of 'analysis of means', discussed in this standard, has application to each of these two methods of experimentation. Beginning with k averages (means) of n measurements each, the analysis compares the individual means with the grand average of the k means; this is directly analogous to a control chart. The decision of a significant difference is made by comparison of points representing the k means with lines drawn parallel to the line of the grand mean. However, instead of control limits drawn at:

$$\bar{x} \pm A_1 \bar{s} \text{ or } \bar{x} \pm A_2 \bar{R}$$

as in the Shewhart Control Chart (where k is usually greater than 20), decision lines for the Analysis of Means are drawn at:

$$\bar{x} \pm H_\alpha \hat{\sigma}_{\bar{x}}$$

where H_α is a factor depending upon level of significance, number of samples examined simultaneously, and degrees of freedom corresponding to the estimate of standard deviation. The values of H_α are given in Appendix A for 5 percent and 1 percent level of significance.

3.4 A chart is constructed using the decision lines as $\bar{x} \pm H_\alpha \hat{\sigma}_{\bar{x}}$. If all the plotted points fall between the upper and lower decision lines, it is interpreted as representing only random variability, that is, no significant difference. If a point falls outside of either of these decision lines, it is considered evidence of assignable cause with risk α .

4. ADVANTAGES OF GRAPHICAL ANALYSIS OF MEANS

4.1 It provides a direct study of possible effects of the factors by dealing with means instead of variances. The analysis of means thus provides a comparison of the relative importance and magnitude of the factors, as well as their statistical significance.

4.2 It provides a graphical comparison of effects. A primary objective of industrial experimentation is not only to obtain information but to present it in a way which will be accepted as the basis for decision and action by appropriate technical and administrative personnel. The graphical presentation facilitates in translating the conclusions into scientific action.

4.3 It is helpful in pin-pointing the sources of assignable variation. An analysis of variance may indicate certain factors which affect the response being studied which must usually be followed by some supplementary analysis to pin-point the important factors. The technique of analysis of means provides such a supplementary analysis also.

4.4 A graphical presentation of data is almost a necessity for interpreting the meaning of interactions whose presence have been indicated by an analysis of variance.

4.5 The 'analysis of means' is more sensitive in detecting the assignable cause of a single mean than analysis on variance. Many industrial studies comparing the performance of several machines, heads or operators indicate that it is important to have methods of detecting a difference in behaviour of one or two (conversely, the analysis of means is somewhat less sensitive in determining the overall variability of a group of machines, heads, or operators).

4.6 The graphical procedure of analysis of means frequently provides a bonus by suggesting the unsuspected presence of certain types of assignable causes; these suggestions can then be included in subsequent experiments for study.

5. DECISION LINES

5.1 Standard Values Known — This condition is represented by a process with long record of stability or control with known average (μ) and standard deviation (σ). The process may be a laboratory procedure where control of independent variables may be relatively easy, or it may be a production process. k independent samples, each of size n are selected from the process and the means of all the k independent samples are examined simultaneously. The interval about mean so that all the k means will lie within this interval with a probability $(1 - \alpha)$ is given as:

$$\mu \pm Z_\alpha \sigma_{\bar{x}} \text{ (where } \sigma_{\bar{x}} = \sigma/\sqrt{n} \text{)}$$

where Z_α is a factor which depends upon the level of significance and the number of samples examined simultaneously. The values of this factor are given in Appendix B for 5 percent and 1 percent level of significance.

5.1.1 It should be noted that when 'control limits' are drawn, for example, at $\mu \pm 2 \sigma_{\bar{x}}$, it is commonly believed that a point outside these limits indicates an assignable cause with risk about 0.05 (5 percent). The risk is indeed about 0.05 provided the criteria is applied to single point just observed; but it is an appreciably greater risk when applied to $k > 1$ means, simultaneously. If $k = 10$, for example, the risk is: $1 - (0.954)^{10} = 0.376$. This is not even close to a five percent risk. Thus, it should be emphasized that the analysis of means given in this standard provides correct probabilities for the analysis of groups of data. Monitoring a process by examining means, one at a time as they become available would, of course, require limit lines set for individual means.

5.1.2 The values in Appendix B thus indicate where to draw decision lines in order to provide an overall risk of α percent when a group of k means is to be analyzed. The procedure for calculation of values of Z_α is given in B-2.

5.2 Standard Values not Known — When the standard values of the process average and standard deviation are not known, the measurements for k independent random samples drawn from the process are obtained and their means (\bar{x}) are computed. Instead of a given mean μ , the grand average $\bar{\bar{x}}$ of the k average is used and instead of a known standard deviation, its estimate $\hat{\sigma}$ is obtained. The decision lines, in this case, are obtained as follows:

$$\text{Central line (CL)} = \bar{\bar{x}}$$

$$\text{Upper decision line (UDL)} = \bar{\bar{x}} + H_\alpha \hat{\sigma}_{\bar{x}}$$

$$\text{Lower decision line (LDL)} = \bar{\bar{x}} - H_\alpha \hat{\sigma}_{\bar{x}}$$

5.2.1 The values of H_α for $\alpha = 5$ percent and 1 percent for selected values of k and degrees of freedom are given in Appendix A.

5.2.2 Standard deviation may be estimated from the average range of k independent random samples drawn from the process. When k samples, each of the same size are examined, and their range is calculated, the average range (\bar{R}) is then obtained from these ranges. The homogeneity of the ranges is then checked with the help of usual control limits for range, that is, $UCL = D_4 \bar{R}$ and $LCL = D_3 \bar{R}$. Once the ranges are homogenized, the average range (\bar{R}) is calculated and then the estimate of standard deviation is given by:

$$\hat{\sigma} = \frac{\bar{R}}{d_2^*}$$

The values of d_2^* which depend upon the number of samples and size of each sample, are given in Appendix C.

NOTE — The standard control chart procedure for estimating standard deviation is given as $\hat{\sigma} = \bar{R}/d_2$, where d_2 is a constant which is independent of k . The value of k is usually as large as 20 or more in most control chart work. In this standard, factor d_2^* is used instead of d_2 , especially when k is small. The values of d_2^* are essentially independent of k and equal to d_2 provided k is large (more than 20). An adjustment is made for small values of k . Actually, use of the d_2 factor to obtain $\hat{\sigma}$ gives a slightly larger estimate since values of d_2^* decrease as k increases. In practice, therefore, some significant differences may be missed if d_2 is used instead of d_2^* .

5.2.3 The computational procedures for both the analysis of means and analysis of variance require reference to the number of degrees of freedom. The exact number of degrees of freedom associated with the estimate of standard deviation based on the range are given in Appendix C for few sample sizes which in general can be approximated by the following relation:

$$\text{Degrees of freedom} \cong 0.90 (n - 1) k.$$

6. ANALYSIS OF MEANS FOR 2^m FACTORIAL DESIGNS

6.1 The various steps to be followed in the analysis of means for 2^m factorial designs, that is, m independent variables each at two levels, with r replicates are as follows:

- a) Obtain and plot the range values for each of the 2^m combinations. Find mean range (\bar{R}) and $D_4 \bar{R}$ and use the range chart as a check on possible outliers.
- b) Compute $\hat{\sigma} = \bar{R}/d_2^*$ where the values of d_2^* are given in Appendix C.
- c) Degrees of freedom are $\cong 0.9 k (r - 1) \cong 0.9 \times 2^m \times (r - 1)$ associated with $\hat{\sigma}$.
- d) Let the independent variables be denoted as A, B, C, \dots ; The lower and the upper level of variable A be denoted as A_1 and A_2 . Similarly, the two levels for the other variables may be defined.
- e) Compute \bar{A}_1 which is the average of all observations performed under experimental condition, A_1 (different levels of remaining variables). Similarly, \bar{A}_2 is defined as average of all observations performed under experimental condition, A_2 . The average responses for the other variables may be defined in similar way.
- f) Calculate overall average as follows:

$$\bar{\bar{x}} = \frac{\bar{A}_1 + \bar{A}_2}{2} = \frac{\bar{B}_1 + \bar{B}_2}{2} = \dots$$

g) Obtain the value of H_α from Appendix A for $l = 2$ (number of levels of each factor), degrees of freedom for $\hat{\sigma}$ and level of significance.

h) Compute and draw the lines at:

$$UDL = \bar{x} + H_\alpha \hat{\sigma}_{\bar{x}}$$

$$LDL = \bar{x} - H_\alpha \hat{\sigma}_{\bar{x}}$$

Any point outside decision lines indicate a statistically significant difference with a risk α percent.

6.2 Example — In order to improve certain quality problems during production of batteries, an experiment with three independent variables (factors) was conducted as follows:

- A_1 : Using one nitrate concentration,
- A_2 : Using different nitrate concentration,
- B_1 : Using a shim in the battery cells,
- B_2 : Not using a shim,
- C_1 : Using fresh hydroxide, and
- C_2 : Using reused hydroxide.

For each subgroup, six batteries ($r = 6$) were tested for capacitance and the test results are given in Table 1. Test if the three main effects and interaction AB are significant.

6.2.1 The various steps for the analysis are as follows:

a) $\bar{R} = \frac{\Sigma R}{k} = \frac{11.6}{8} = 1.45$ where k is total number of subgroups

$$D_4 \bar{R} = 2.004 \times 1.45 = 2.90$$

Since all range points fall below $D_4 \bar{R}$, \bar{R} is taken as homogenized value of the range.

b) $\bar{R}/d_2^* = \frac{1.45}{2.55} = 0.57$

$$\hat{\sigma}_{\bar{x}} = \frac{\hat{\sigma}}{\sqrt{24}} = 0.12$$

c) Degrees of freedom $\cong 0.9 \times 2^3 \times (6 - 1) \cong 36$

d) $\bar{A}_1 = \frac{\text{Sum of columns 1, 2, 3, and 4}}{\text{Number of observations}} = \frac{15.4}{24} = 0.64$

Similarly, $\bar{A}_2 = \frac{\text{Sum of columns 5, 6, 7 and 8}}{\text{Number of observations}} = 1.73.$

In this way, the average response for each level of the factors is calculated and the values are given in Table 2.

TABLE 1 CAPACITANCE OF INDIVIDUAL BATTERIES (DATA CODED)

A_1				A_2				
B_1		B_2		B_1		B_2		
C_1	C_2	C_1	C_2	C_1	C_2	C_1	C_2	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	
- 0.1	1.1	0.6	0.7	0.6	1.9	1.8	2.1	
1.0	0.5	1.0	- 0.1	0.8	0.7	2.1	2.3	
0.6	0.1	0.8	1.7	0.7	2.3	2.2	1.9	
- 0.1	0.7	1.5	1.2	2.0	1.9	1.9	2.2	
- 1.4	1.3	1.3	1.1	0.7	1.0	2.6	1.8	
0.5	1.0	1.1	- 0.7	0.7	2.1	2.8	2.5	
TOTAL	0.5	4.7	6.3	3.9	5.5	9.9	13.4	12.8
\bar{x}	0.08	0.78	1.05	0.65	0.92	1.65	2.23	2.13
R	2.4	1.2	0.9	2.4	1.4	1.6	1.0	0.7

TABLE 2 AVERAGE RESPONSE OF MAIN EFFECTS AND INTERACTION AB

[Clause 6.2.1(d)]

$\bar{A}_1 = 0.64$	$\bar{B}_1 = 0.86$	$\bar{C}_1 = 1.07$	$(AB)_L = 1.30$
$\bar{A}_2 = 1.73$	$\bar{B}_2 = 1.52$	$\bar{C}_2 = 1.30$	$(AB)_U = 1.06$

e) For obtaining average response for two-factor interactions AB , the following procedure is adopted:

$(AB)_L$ = Average response at lower level of AB is equal to sum of the values of all columns where AB is either at first level (A_1B_1) or at second level (A_2B_2) divided by the number of observations

$$= \frac{\text{Sum of columns 1, 2, 7 and 8}}{24}$$

$$= 1.305$$

$(AB)_U$ = The average response at upper level of AB is equal to sum of the values of all columns having combinations A_1B_2 or A_2B_1 divided by the number of observations

$$= \frac{\text{Sum of columns 3, 4, 5 and 6}}{24}$$

$$= 1.065$$

f) The overall average is calculated as:

$$\bar{x} = \frac{A_1 + A_2}{2} = 1.185$$

g) The values of H_α from Appendix A for $k = 2$ (number of levels of each factor) and degrees of freedom = 36 for 5 percent and 1 percent level of significance are as follows:

For $\alpha = 0.05$, $H_\alpha = 1.43$, and

For $\alpha = 0.01$, $H_\alpha = 1.92$.

h) Compute the decision lines as follows:

$$\begin{aligned} \text{UDL (0.05)} &= 1.185 + (1.43) (0.12) \\ &= 1.185 + 0.17 = 1.35 \end{aligned}$$

$$\text{LDL (0.05)} = 1.185 - 0.17 = 1.02$$

$$\text{UDL (0.01)} = 1.185 + 0.223 = 1.41$$

$$\text{LDL (0.01)} = 1.185 - 0.223 = 0.96$$

6.2.2 The decision lines as calculated in 6.2 and also the average responses for the factors are plotted (see Fig. 1).

6.2.3 Conclusions — Figure 1 indicates that A and B main effects are highly significant (that is, $\alpha < 0.01$). Combinations given in column 7 and 8 with A_2 and B_2 evidently are the best as they give higher values of capacitance. Besides the demonstrated advantages of A_2 over A_1 , and of B_2 over B_1 , it has been concluded that the main effect C has no significant difference. This result indicates that a certain expensive hydroxide may be reused in manufacturing. The interaction between factors A and B is also not significant.

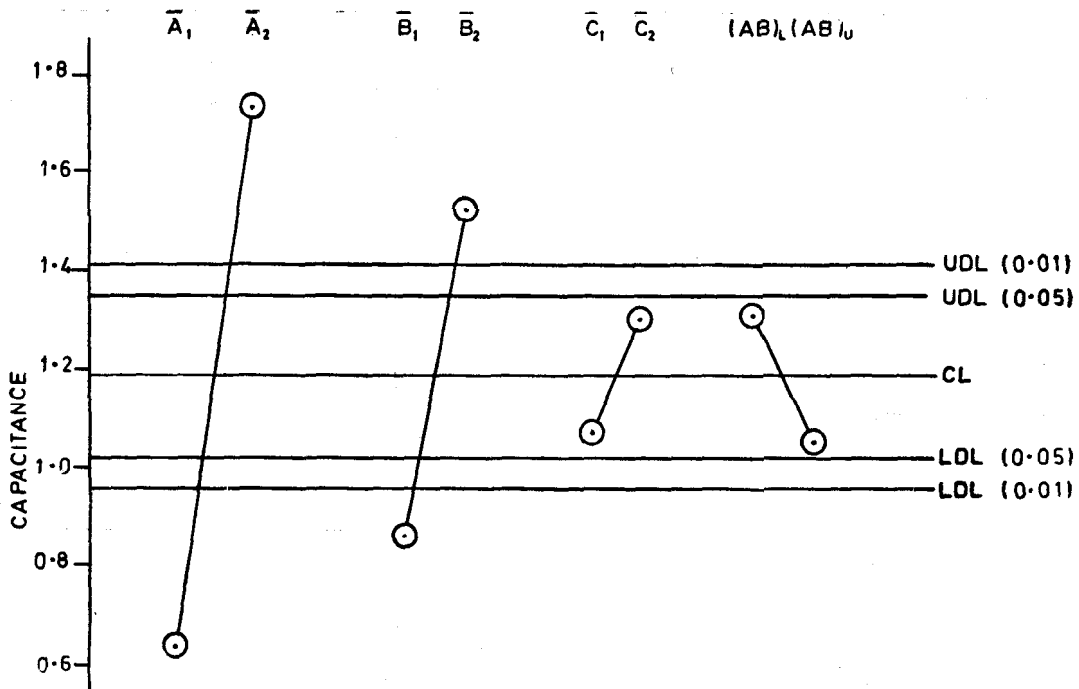


FIG. 1 CONTROL CHART

7. PROCEDURE FOR ANALYSIS OF MEANS FOR VARIABLES AT DIFFERENT LEVELS

7.1 The various steps to be followed in the analysis of means when there is more than one independent variable, each having two or more levels with r replicates for each subgroup are as follows:

- a) Obtain and plot the range values for all the k subgroups. Find mean range (\bar{R}) and $D_4\bar{R}$. Use the range chart as a check on possible outliers.
- b) Compute $\hat{\sigma} = \bar{R}/d_2^*$ where the values of d_2^* are given in Appendix C.
- c) Degrees of freedom associated with estimate of standard deviation are $\cong 0.9 k (r - 1)$.
- d) Let the independent variables be:
 X and let its levels be X_1, X_2, \dots, X_g , and
 Y and let its levels be Y_1, Y_2, \dots, Y_h , etc.
- e) Compute $\bar{X}_1, \bar{X}_2, \dots, \bar{X}_g$ where each \bar{X}_1 is the average of all observations performed under experimental conditions X_1 (at all different levels of Y and any other independent variables). The number of individual observations in \bar{X}_1 will be $N/g = k_x$ where N is the total number of individual observations. Similarly, \bar{y}_1 will be the average of $k_y = N/h$ individual observations, etc.

- f) Plot the averages \bar{X}_1 on one chart and the \bar{y}_1 averages on the same chart extended.
- g) Compute and draw decision lines (main effects):

for \bar{X}_1 using $\hat{\sigma}_{\bar{x}} = \hat{\sigma} / \sqrt{N/g}$ and for \bar{y}_1 using $\hat{\sigma}_{\bar{y}} = \hat{\sigma} / \sqrt{N/h}$.

Find $H_{0.05}$ from Appendix A corresponding to degrees of freedom = $0.9 k (r - 1)$ and $k = k_x$ for the \bar{X}_1 and $k = k_y$ for the \bar{Y}_1 .

- h) Compute and draw decision lines at:

$\bar{X} \pm H_{\alpha} \hat{\sigma}_{\bar{x}}$ and $\bar{Y} \pm H_{\alpha} \hat{\sigma}_{\bar{y}}$

where $\bar{X} = \bar{Y} = \bar{C}$.

Any point outside decision lines indicates a statistically significant difference from the group average.

7.2 Example — Table 3 gives the lengths of the steel bars (coded) which were made from two heat treatments (A, B), cut on three screw machines (M_1, M_2, M_3) and at three different times (T_1, T_2, T_3). The time element involved fatigue on the part of operator. Find, using technique of analysis of means, if any of the factors are significant. Also test significance of interaction between heat treatments and machines.

TABLE 3 LENGTHS OF STEEL BARS (CODED)

TIME	HEAT TREATMENT (A)			HEAT TREATMENT (B)		
	Machine			Machine		
	M_1	M_2	M_3	M_1	M_2	M_3
(1)	(2)	(3)	(4)	(5)	(6)	(7)
	5	7	2	4	8	-1
	10	9	2	3	5	2
T_1	2	6	1	2	3	0
	3	8	4	6	4	1
Average	5.0	7.5	2.2	3.8	5.0	0.5
Range	8	3	3	4	5	3
	4	9	4	3	4	2
	3	6	2	4	0	2
T_2	-1	5	1	1	5	-1
	1	8	5	-2	3	1
Average	1.8	7.0	3.0	1.5	3.0	1.0
Range	5	4	4	6	5	3
	4	10	-1	6	6	3
	5	7	2	1	7	-2
T_3	1	6	6	3	10	3
	6	5	3	8	2	-4
Average	4.0	7.0	2.5	4.5	6.2	0.0
Range	5	5	7	7	8	7

7.2.1 The various steps in the analysis are as follows:

- a) $\Sigma R = 92, \bar{R} = \frac{\Sigma R}{18} = \frac{92}{18} = 5.11$
 $r =$ number of replicates in each sub-group $= 4,$
 $D_4 = 2.28$
 $D_4\bar{R} = 2.28 \times 5.11 = 11.66$
 Since all the range values are below $D_4\bar{R}, \bar{R}$ is taken as the homogenized value of the range.
- b) For $r = 4$ and $k = 18, d_2^* = 2.07$ (see Appendix C)
 $\hat{\sigma} = \frac{\bar{R}}{d_2^*} = \frac{5.11}{2.07} = 2.47$
- c) As the standard deviation is estimated using the range method with $k = 18$ and $r = 4,$ the degrees of freedom $= 0.90 \times 18 \times (4 - 1) = 49.$
- d) The summary of average responses of main effects is given in Table 4.

TABLE 4 AVERAGE RESPONSES OF MAIN EFFECTS

HEAT (1)	MACHINE (2)	TIME (3)
$\bar{A} = 4.44$	$\bar{M}_1 = 3.42$	$\bar{T}_1 = 4.00$
$\bar{B} = 2.83$	$\bar{M}_2 = 5.96$	$\bar{T}_2 = 2.88$
	$\bar{M}_3 = 1.54$	$\bar{T}_3 = 4.04$
$n_h = \frac{72}{2} = 36$	$n_r = \frac{72}{3} = 24$	$n_t = \frac{72}{3} = 24$

e) $\bar{x} = \frac{\bar{M}_1 + \bar{M}_2 + \bar{M}_3}{3} = \frac{\bar{T}_1 + \bar{T}_2 + \bar{T}_3}{3}$
 $= \frac{\bar{A} + \bar{B}}{2} = 3.64$

f) $\hat{\sigma}_h = \hat{\sigma} / \sqrt{n_h} = 2.47 / \sqrt{36} = 0.412$
 $\hat{\sigma}_m = \frac{2.47}{\sqrt{24}} = 0.504 = \hat{\sigma}_t$

- g) $k_h =$ number of levels of heat treatment $= 2$
 Similarly $k_m = k_t = 3$
 The values of H_{α} for given k and degrees of freedom $= 49$ are as follows:

$k_h = 2 \quad k_m = k_t = 3$
 $H_{0.05} = 1.42 \quad H_{0.05} = 2.00$
 $H_{0.01} = 1.90 \quad H_{0.01} = 2.51$

- h) The decision lines for the main effects are as follows:

Heat

UDL (0.05) $= \bar{x} + H_{0.05} \hat{\sigma}_h$
 $= 3.64 + 1.42 (0.412) = 4.23$
 LDL (0.05) $= \bar{x} - H_{0.05} \hat{\sigma}_h$
 $= 3.64 - 0.585 = 3.06$
 UDL (0.01) $= \bar{x} + H_{0.01} \hat{\sigma}_h$
 $= 3.64 + 1.90 (0.412) = 4.42$

LDL (0.01) $= \bar{x} - H_{0.01} \hat{\sigma}_h = 2.86$

Machine and Time

As $\hat{\sigma}_m = \hat{\sigma}_t = 0.504$ and values of H_{α} are also the same, the decision lines will be same for both the main effects and are given below:

UDL (0.05) $= \bar{x} + H_{0.05} \hat{\sigma}_m = 4.65$
 LDL (0.05) $= 2.63$
 UDL (0.01) $= 4.91$
 LDL (0.01) $= 2.37$

- j) The decision lines and average responses are plotted on graph (see Fig. 2). From this figure, it is clear that the differences among machines contribute most to variability in length of the steel bars. The effect of heat treatment is also significant. The magnitude of machine differences is greater than the magnitude of heat treatment differences. Time factor does not show a statistically significant effect.
- k) Two-factor interaction — Averages are found ignoring all factors except the two being considered. For example, in testing the significance of interaction between heat treatments and machines, the time is ignored and the averages of three machines at heat A and B are calculated (see Table 5). If the average differences Δ_1 (see Table 5) represent random variation about their own average $\bar{\Delta}$, there is no interaction.

TABLE 5 AVERAGES OF M x H INTERACTION

HEAT TREATMENT	MACHINE		
	M ₁	M ₂	M ₃
(1)	(2)	(3)	(4)
A	3.58	7.17	2.58
B	3.25	4.75	0.50
$\Delta_1 = A - B$	0.33	2.42	2.08

The procedure adopted is to calculate decision lines for Δ_1 as follows:

$\bar{\Delta} = \frac{\Sigma \Delta_1}{3} = 1.61$

$\hat{\sigma}_{\Delta} = \sqrt{2} \hat{\sigma}_{\bar{x}} = \sqrt{2} \times \frac{2.47}{\sqrt{12}} = 1.01$

For $k = 3$ and degrees of freedom $= 49, H_{0.05} = 2.00$ and $H_{0.01} = 2.51$

UDL (0.05) $= \bar{\Delta} + H_{0.05} \hat{\sigma}_{\Delta} = 1.61 + 2.00 (1.01) = 3.63$
 LDL (0.05) $= -0.41$

Since all the three points of Δ_1 are within the two decision lines, it is concluded that there is no significant $M \times H$ interaction (see Fig. 3).

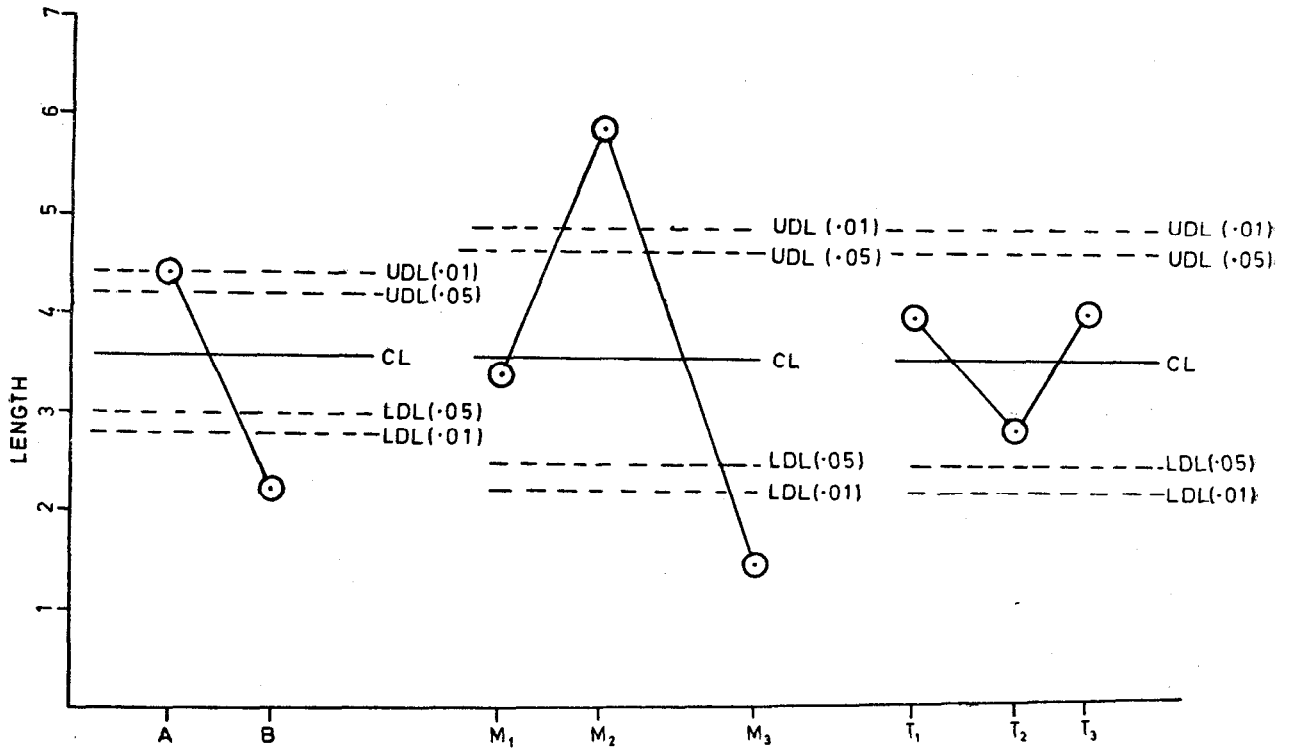


FIG. 2 MAIN EFFECTS ANALYSIS

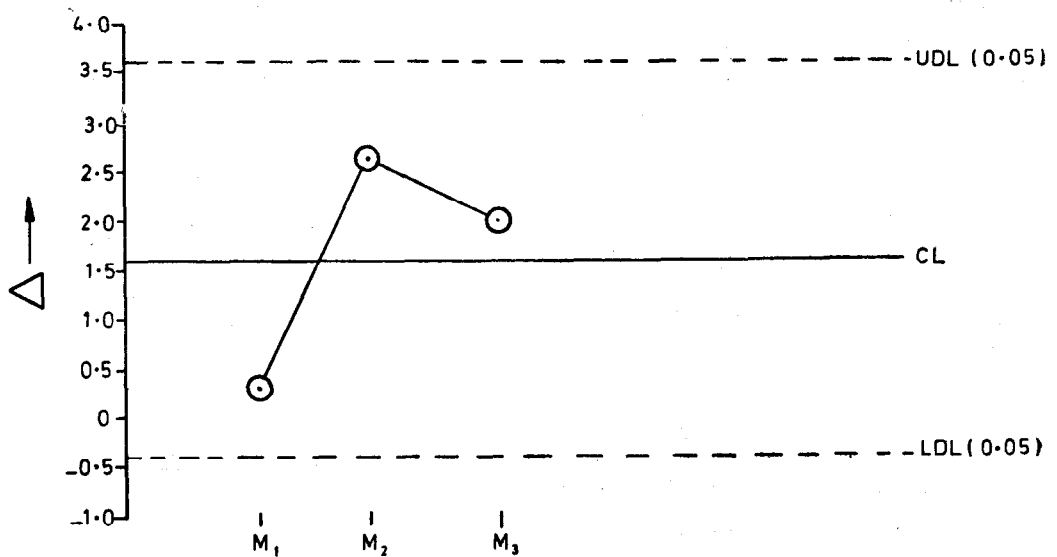


FIG. 3 INTERACTION ANALYSIS

APPENDIX A
(Clauses 3.3 and 5.2.1)

VALUES OF H_α

Degrees of Freedom	Number of Means Being Compared (k)									
	2	3	4	5	6	8	10	15	20	
$\alpha = 0.05$										
5	1.82									
6	1.73	2.59	2.94	3.19	3.37					
8	1.63	2.39	2.71	2.92	3.09	3.33				
10	1.58	2.29	2.58	2.78	2.93	3.15	3.31			
15	1.51	2.16	2.42	2.60	2.74	2.93	3.07	3.32		
20	1.48	2.10	2.35	2.52	2.64	2.83	2.96	3.18	3.33	
30	1.44	2.04	2.28	2.44	2.56	2.73	2.86	3.06	3.19	
40	1.43	2.01	2.25	2.40	2.52	2.69	2.80	3.00	3.13	
60	1.41	1.98	2.21	2.36	2.48	2.64	2.76	2.94	3.06	
120	1.40	1.95	2.18	2.33	2.44	2.60	2.71	2.88	3.00	
α	1.39	1.93	2.15	2.29	2.40	2.55	2.65	2.82	2.94	
$\alpha = 0.01$										
5	2.85									
6	2.62	3.74	4.21	4.53	4.78					
8	2.37	3.31	3.70	3.97	4.17	4.47				
10	2.24	3.08	3.43	3.67	3.86	4.11	4.29			
15	2.08	2.81	3.12	3.32	3.47	3.69	3.84	4.11		
20	2.01	2.70	2.98	3.17	3.30	3.50	3.63	3.87	4.02	
30	1.94	2.58	2.85	3.02	3.15	3.33	3.45	3.66	3.79	
40	1.91	2.53	2.79	2.95	3.07	3.24	3.36	3.56	3.68	
60	1.88	2.48	2.73	2.88	3.00	3.16	3.27	3.46	3.58	
120	1.85	2.43	2.67	2.82	2.93	3.09	3.20	3.37	3.48	
α	1.82	2.39	2.61	2.76	2.87	3.02	3.12	3.29	3.39	

APPENDIX B
(Clauses 5.1 and 5.1.2)

B-1. VALUES OF Z_α

Number of Means (k)	$\alpha = 5$ percent	$\alpha = 1$ percent	Number of Means (k)	$\alpha = 5$ percent	$\alpha = 1$ percent
2	2.24	2.81	15	2.93	3.40
3	2.39	2.93	20	3.02	3.48
4	2.49	3.02	24	3.07	3.53
5	2.57	3.09	25	3.08	3.54
6	2.63	3.14	30	3.14	3.59
7	2.68	3.19	40	3.22	3.66
8	2.73	3.23	60	3.28	3.72
9	2.77	3.26	120	3.52	3.93
10	2.80	3.29			

B-2. BASIC CONCEPTS IN DERIVATION OF Z_α

B-2.1 Let p represent the probability that any one mean will lie between decision lines drawn at the previously designated values of:

$$\bar{x} \pm Z_\alpha \hat{\sigma}_{\bar{x}} \quad \dots(1)$$

Then the probability that all k means will lie between the lines given by equation (1) is p^k , and the probability that at least one point will fall outside the lines is $1 - p^k$. Since the decision lines within which all k points will fall with risk α , the value of p can be found from:

$$1 - p^k = \alpha \text{ or } p^k = 1 - \alpha \quad \dots(2)$$

or $k \log p = \log (1 - \alpha)$

B-3. Hence for given values of k and α , the value of p is obtained. Corresponding to the value of p obtained from this equation, the value of Z is located in a table of areas under the normal curve such that the fraction of the area lying between the ordinates given by equation (1) is p .

Example:

Suppose $k = 3$ and $\alpha = 0.05$. Decision lines are required such that one point outside the lines shall be evidence (at the 0.05 level) of non-randomness of the means.

Here

$$\begin{aligned} p^3 &= 0.95 \\ \log p &= (1/3) \log (0.95) \\ \log p &= (1/3) (2.977\ 72 - 3) \\ p &= 0.983\ 04 \end{aligned}$$

The value of Z is then found from a table of areas under the normal curve so that:

$$\begin{aligned} \text{Prob} \left\{ \bar{x} - Z \hat{\sigma}_{\bar{x}} < \mu < \bar{x} + Z \hat{\sigma}_{\bar{x}} \right\} \\ = 0.983\ 04 \end{aligned}$$

which comes out to be $Z = 2.39$.

APPENDIX C

[*Clauses 5.2.2, 5.2.3, 6.1, 7.1(b) and 7 2.1(b)*]

VALUES OF d_2^* FOR ESTIMATING STANDARD DEVIATION FROM k INDEPENDENT SAMPLE RANGES OF n EACH AND ASSOCIATED DEGREES OF FREEDOM

k	$n = 2$		$n = 3$		$n = 4$		$n = 5$		$n = 6$	
	d_1	d_2^*	d_1	d_2^*	d_1	d_2^*	d_1	d_2^*	d_1	d_2^*
1	1.0	1.41	2.0	1.91	2.9	2.24	3.8	2.48	4.7	2.67
2	1.9	1.28	3.8	1.81	5.7	2.15	7.5	2.40	9.2	2.60
3	2.8	1.23	5.7	1.77	8.4	2.12	11.1	2.38	13.6	2.58
4	3.7	1.21	7.5	1.75	11.2	2.11	14.7	2.37	18.1	2.57
5	4.6	1.19	9.3	1.74	13.9	2.10	18.4	2.36	22.6	2.56
6	5.5	1.18	11.1	1.73	16.6	2.09	22.9	2.35	27.1	2.56
7	6.4	1.17	12.9	1.73	19.4	2.09	25.6	2.35	31.5	2.55
8	7.2	1.17	14.8	1.72	22.1	2.08	29.3	2.35	36.0	2.55
9	8.1	1.16	16.6	1.72	24.8	2.08	32.9	2.34	40.5	2.55
10	9.0	1.16	18.4	1.72	27.6	2.08	36.5	2.34	44.9	2.55
11	9.9	1.16	20.2	1.71	30.3	2.08	40.1	2.34	49.4	2.55
12	10.8	1.15	22.0	1.71	33.0	2.07	43.7	2.34	53.9	2.55
13	11.6	1.15	23.9	1.71	35.7	2.07	47.4	2.34	58.4	2.55
14	12.5	1.15	25.7	1.71	38.5	2.07	51.0	2.34	62.8	2.54
15	13.4	1.15	27.5	1.71	41.2	2.07	54.6	2.34	67.3	2.54
16	14.3	1.15	29.3	1.71	43.9	2.07	58.2	2.34	71.8	2.54
17	15.2	1.15	31.1	1.71	46.7	2.07	61.8	2.34	76.2	2.54
18	16.0	1.15	33.0	1.71	49.4	2.07	65.5	2.33	80.7	2.54
19	16.9	1.14	34.8	1.70	52.2	2.07	69.1	2.33	85.2	2.54
20	17.8	1.14	36.6	1.70	54.9	2.07	72.7	2.33	89.7	2.54

$$df \cong 0.9 k (n - 1)$$

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